Reply to Comment by A. Moroz

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In his comment, Moroz questions the validity of the near band edge (effective mass) approximation to the total photon density of states (DOS) as a useful representation of the local density of states (LDOS) experienced by a single radiating atom or molecule located at a particular position $\vec{r}$ within a photonic crystal (PC). In this approximation, the band edge DOS takes the form:

$$\rho(\omega) \approx \text{const}|\omega - \omega_\text{c}|^\eta$$

where $\eta = -0.5$ for a 1-d PC and $\eta = 0.5$ for a 3-d PC. We reassert that this behaviour indeed applies to the LDOS as well as the DOS. However, the frequency range over which this behaviour is realized depends sensitively on $\vec{r}$. In particular, if $\vec{r}$ is chosen near a node of the electromagnetic field intensity $\left|\vec{E}(\vec{r})\right|^2$, then ω must be chosen very close to ω_c before the asymptotic behaviour is realized. The seemingly arbitrary exponents obtained by Moroz are simply an artifact of fitting the asymptotic form to numerical data for a frequency ω which is not sufficiently close to ω_c at certain positions $\vec{r}$.

We consider precisely the example quoted by Moroz in his comment and assume that the LDOS has the asymptotic form:

$$\rho(\omega, \vec{r}) = \mathcal{K}(\vec{r})|\omega_\text{c} - \omega|^\eta$$

Near the lower band edge of the first photonic band gap ($\omega \lessapprox \omega_\text{c}$) we define $u \equiv 1 - \frac{\omega}{\omega_\text{c}} > 0$. In order to numerically estimate the exponent η, we write:

$$y \equiv \log_{10}\rho = \eta(\log_{10} u + \log_{10}\omega_\text{c}) + \log_{10}\mathcal{K}(\vec{r})$$

Using equations (4) and (7) of Moroz's paper [1] we plot (in Fig. 1a) y as a function of $z \equiv \log_{10} u$ for 8 different positions $\vec{r}$ in the 1-d unit cell of the example quoted in the above comment. The asymptotic behaviour of dy/dz for large negative values of z ($\omega \to \omega_\text{c}$) yields the exponent η (see Fig. 1b). In this model the lower band edge mode intensity vanishes at $x \equiv |\vec{r}| = 0.5$ (center of air region) and has a maximum at $x = 0.0$ (center of dielectric slab). For all cases the asymptotic behaviour ($\omega \to \omega_\text{c}$) yields the common exponent $\eta = -0.5$. However arbitrary values of dy/dz, and hence η, may be erroneously inferred by choosing too large a value of $|\omega - \omega_\text{c}|$. This is particularly evident near the node of the field intensity.

We conclude that although the LDOS is sensitive to the actual position $\vec{r}$, the exponent η is indeed universal except on a set of measure zero, namely the field intensity nodes. The seemingly arbitrary exponents quoted by Moroz are somewhat misleading. On the other hand, inhomogeneous line broadening is a very important and relevant ingredient which must be incorporated into theoretical models which aim to interpret experiments involving a distribution of atoms in a PC.

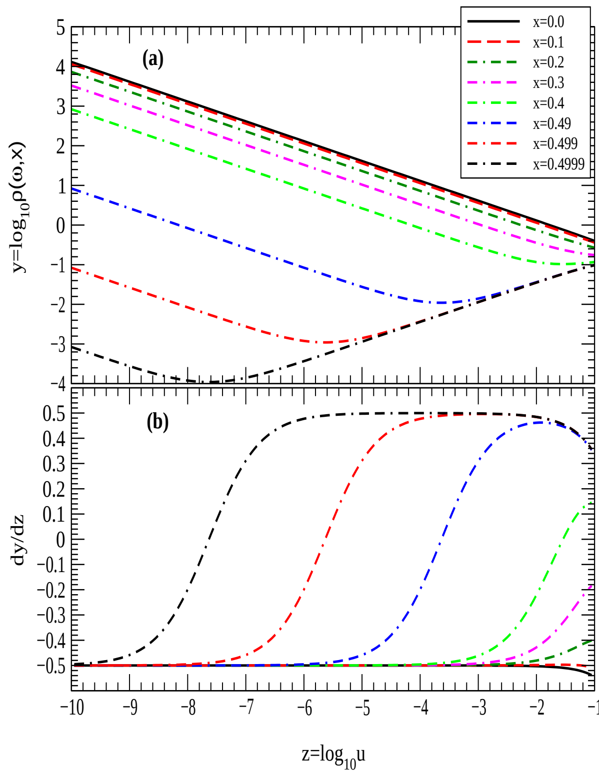

FIG. 1. Top picture shows $\log_{10}\rho(\omega, x)$ as a function of $z = \log_{10} u$ for 8 positions in the unit cell. Here $x = |\vec{r}|$. Bottom picture shows the slope of the curves. In all cases the asymptotic behaviour ($\omega \to \omega_\text{c}$) yields $\eta = -0.5$.

[1] A Moroz, Europhysics Letters **46** (4), 419 (1999).